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LABORATORY FOR INFORMATION AND DECISION SYSTEMS
Massachusetts Institute of Technology
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Annual Report for

Grant AFOSR-82-0258

on

ANALYSIS, ESTIMATION, AND CONTROL FOR
PERTURBED AND SINGULAR SYSTEMS AND
FOR SYSTEMS SUBJECT TO DISCRETE EVENTS

for the period

1 July 1986 through 30 September 1987

Submitted to: Major James Crowley
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I. SUMMARY

In this report we summarize our accomplishments in the research program presently supported by Grant AFOSR-82-0258 over the period from July 1, 1982 to September 30, 1987, with primary emphasis on the accomplishments from July 1, 1986 to September 30, 1987. The basic scope of this program is the analysis, estimation, and control of complex systems with particular emphasis on (a) the development of asymptotic methods and theories for nearly singular systems; (b) the investigation of theoretical questions related to singular systems; and (c) the analysis of complex systems subject to or characterized by sequences of discrete events. These three topics are described in the next three sections of this report. A full list of publications supported by Grant AFOSR-82-0258 is also included.

The principal investigator for this effort is Professor Alan S. Willsky, and Professor George C. Verghese is co-principal investigator. Professors Willsky and Verghese were assisted by several graduate research assistants as well as additional thesis students not requiring stipend or tuition support from this grant. The list of 47 publications includes 14 papers that have appeared or have been submitted to journals, 9 journal papers presently in preparation, 14 papers presented at conferences, 1 S.B. thesis, 3 S.M. theses, and 6 Doctoral theses. In addition, Prof. Willsky and Verghese have been invited to give a number of lectures on the results of these efforts including Prof. Willsky's featured invited presentation at the August 1986 SIAM Conference on Linear Algebra in Signals, Systems and Control.

2

II. ASYMPTOTIC ANALYSIS FOR PERTURBED SYSTEMS

Our previous research in this general area has produced a number of important results and directions for further research. In this subsection we review the basic ideas behind our work which is documented in detail in[1] [1-4, 7, 9, 11-13, 16, 20-21, 25-26, 30-32, 34-37, 44 and 47].

The model that has been the focus of much of our attention is the perturbed linear system

$$\dot{x}(t) = A(\epsilon)x(t) \qquad (2.1)$$

where $A(\epsilon)$ is analytic in ϵ at $\epsilon = 0$. If, furthermore, $A(\epsilon)$ loses rank at $\epsilon = 0$, (2.1) represents a singularly perturbed system that may display dynamics at several time scales. Such models arise in describing complex interconnected systems with weak couplings, "stiff" systems with time constants ranging over several orders of magnitude, and finite-state Markov processes (FSMP's) with rare transitions. In this latter case $A(\epsilon)$ is an infinitessimally stochastic matrix (i.e., column sums are zero and off-diagonal terms are nonnegative) and $x(t)$ is the vector of state probabilities.

Our earliest work [1, 2, 4] on analyzing (2.1) used results on perturbations of linear operators [Kato 1982] to develop a general approach to determining if (2.1) has well-behaved time scale structures and, if so, to

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[1]In this report we refer to publications supported by AFOSR by number, e.g. [8]. References to other work are included in a second list and are referred to by author and year, e.g. [Kato 1982].

construct a multiple time scale approximation. In the case of FSMP's this work made clear the connection with stochastically discontinuous FSMP's and provided a general result on hierarchical aggregation of perturbed FSMP's. The basic idea behind the approach in [1, 2, 4] is an examination of the perturbed eigenstructure of (2.1). Specifically, let $P_0(\epsilon)$ denote the projection onto the subspace spanned by the eigenvectors and generalized eigenvectors corresponding to eigenvalues of $A(\epsilon)$ that converge to 0 as $\epsilon \downarrow 0$. Then let

$$A_1(\epsilon) = P_0(\epsilon)A(\epsilon)/\epsilon = P_0(\epsilon)A(\epsilon)P_0(\epsilon)/\epsilon \qquad (2.2)$$

As discussed in [1, 2, 4], $A_1(\epsilon)$ is analytic at $\epsilon = 0$ if and only if $A(0)$ has semisimple null-structure (SSNS). In this case the process can be iterated to produced $A_2(\epsilon), A_3(\epsilon)$, etc. If this procedure can be taken to completion, $A(\epsilon)$ is said to have multiple semisimple null-structure (MSSNS), and if $A(0), A_1(0), A_2(0), \ldots$ are all semistable (i.e., all eigenvalues strictly in the left-half plane except for possible semisimple zero eigenvalues), $A(\epsilon)$ is said to satisfy the multiple semistability (MSST) condition. In this case the dynamics in (2.1) can be uniformly approximated by $A(0), \epsilon A_1(0), \ldots$ in the sense that

$$\lim_{\epsilon \downarrow 0} \sup_{t \geq 0} \| \exp\{A(\epsilon)t\} - e^{A(0)t} \, e^{A_1(0)\epsilon t} \, e^{A_2(0)\epsilon^2 t} \ldots \| = 0 \qquad (2.3)$$

Furthermore, if $A(\epsilon)$ is infinitessimally stochastic, it is possible after the fact to represent each successive time scale in (2.3) in terms of an aggregated version of the FSMP at the preceding time scale.

4

While these results are quite general, the price that apparently is paid for this generality is a significant increase in complexity and a corresponding loss of simple interpretation when compared to other results developed for restricted classes of systems. In particular, the method in [1, 2, 4] requires the computation of the entire ϵ-dependent projection $P_0(\epsilon)$, even though the ultimate objective, as shown in (2.3), is to discard all but the critical ϵ-dependencies (as embodied in the matrices $A(0), \epsilon A_1(0), \epsilon^2 A_2(0), \ldots$). Consequently a key thrust of our subsequent research has been to provide a bridge between our general results and previous simpler ones in order both to develop alternate, simpler procedures and to pinpoint the precise causes of increased complexity in the general case.

In [7, 9 11-13, 20-21, 26], we have exposed the importance of the invariant factors of $A(\epsilon)$, viewed as a matrix over the ring of functions of ϵ analytic at $\epsilon = 0$. Specifically, consider the Smith decomposition of $A(\epsilon)$:

$$A(\epsilon) = P(\epsilon)D(\epsilon)Q(\epsilon) \tag{2.4}$$

where $|P(0)|, |Q(0)| \neq 0$ and

$$D(\epsilon) = \text{diag}(\epsilon^{k_1} I_1, \ldots, \epsilon^{k_n} I_n) \tag{2.5}$$

where the ϵ^{k_i} are the invariant factors of $A(\epsilon)$. Then, as shown in [20], the time-scale analysis of $A(\epsilon)$ is equivalent to that for $D(\epsilon)\bar{A}$ where $\bar{A}$ is the ϵ-independent matrix $Q(0)P(0)$. In taking this step we have discarded a significant number of ϵ-dependent terms and have put the system into an explicit form that allows us to make direct contact with previous results. In particular, the time scales of the system, if they exist, are precisely determined by the invariant factors, and the MSSNS and MSST conditions can be

5

directly related to the properties of a sequence of successive Schur complements of $\bar{A}$. This approach also allows us to make a stronger and more precise statement of the main results in [1] involving in particular the notion of a strong time scale decomposition.

These results prompted additional research on the relationship between the invariant factor structure and eigenstructure of $A(\epsilon)$. In particular, in [9, 11 21] we show that MSSNS is equivalent to the orders of the eigenvalues equalling those of the invariant factors. Going one step farther, note that the gcd of all minors of $A(\epsilon)$ of various orders determine the invariant factors of $A(\epsilon)$, while the sums of principal minors of each order specify the characteristic polynomial of $A(\epsilon)$ and therefore the orders of the eigenvalues. From this observation we find that MSSNS is equivalent to a particular consistency condition among these integer orders together with a "non-cancellation" condition that guarantees that the leading terms of principal minors of particular orders are not canceled when they are summed.

These conditions also suggest a related line of investigation for which we have some initial results [9, 12 26], namely the use of amplitude scaling to modify non-principal minors of $A(\epsilon)$ so that the MSSNS condition is satisfied. Consider, for example, the following system matrix that does not have MSSNS:

$$A(\epsilon) = \begin{bmatrix} -\epsilon & 1 \\ 0 & -\epsilon \end{bmatrix} \tag{2.6}$$

Note that the reason that (2.3) cannot be satisfied is that the (1,2)-element of $\exp\{A(\epsilon)t\}$ is $te^{-\epsilon t}$ which has a maximal value of order $1/\epsilon$. Consider,

however, a similarity transformation that scales the state variables

$$z(t) = \text{diag}(\epsilon, 1) \, x(t) \qquad (2.7)$$

The transformed system matrix in this case is

$$\begin{bmatrix} -\epsilon & \epsilon \\ 0 & -\epsilon \end{bmatrix} \qquad (2.8)$$

which does have MSSNS. The procedure we have developed identifies diagonal scalings for a restricted class of system matrices by identifying those minors of $A(\epsilon)$ that are the reason for the violation of the MSSNS condition. We expect that there is a generalization of this procedure that is applicable to a far larger class of systems. Indeed, we have seen how our procedure can be adapted to recover the special cheap control and high-gain scaling results in [Sannuti 1983], but a more general result remains for the future.

In [16, 25, 30-32, 36-37, 44, 47] we describe a series of results that have arisen out of a second aspect of our efforts to simplify and interpret the results in [1, 2, 4], in this case for FSMP's. As discussed in [16], this line of research was motivated by a desire to understand the relationship of the method in [1, 2, 4] to simpler results such as [Courtois 1977]. Specifically, for an FSMP, the eigenprojection $P_0(\epsilon)$ evaluated at $\epsilon = 0$ is the ergodic projection of the FSMP corresponding to the matrix $A(0)$. Instead of $A_1(\epsilon)$ in (2.2) consider

$$F_1(\epsilon) = P_0(0)A(\epsilon)P_0(0)/\epsilon \qquad (2.9)$$

Note that since $P_0(0)$ is an ergodic projection it can be written as

$$P_0(0) = UV \qquad (2.10)$$

where each column of U is the vector of ergodic probabilities for a single ergodic class of $A(0)$. The matrix V is a membership matrix, with each row specifying which states are in a particular ergodic class. From this one can

7

deduce that VU = I and that

$$\exp\{F_1(\epsilon)t\} = U \exp\{G_1(\epsilon)t\}V \qquad (2.11)$$

where

$$G_1(\epsilon) = VA(\epsilon)U/\epsilon \qquad (2.12)$$

corresponds to an aggregated FSMP with one state corresponding to each ergodic class of the original unperturbed FSMP (characterized by $A(0)$). The rates between these aggregates, as specified by (2.12), represent average transition rates from states in one ergodic class to states in another, with the averaging done using the ergodic probabilities in U.

As pointed out in [16], the procedure just described breaks down if the original FSMP has implicit time scale behavior resulting from the existence of critical _sequences_ of rare transitions from one ergodic class of $A(0)$ to another. Such sequences, which arise naturally in problems such as reliability analysis of complex, fault-tolerant systems and queueing analysis of data communication networks, correspond to the existence of transient states in $A(0)$, and transitions through such states are completely missed by the averaging in (2.12). By keeping all ϵ-dependencies, as in (2.2), we avoid this problem but with a considerable increase in complexity. In contrast, in [16] we describe a method for computing only those ϵ-dependent terms that are critical in describing longer-term behavior. Specifically, this procedure involves replacing the "membership matrix" V in (2.11) by an ϵ-dependent membership matrix. The ϵ-dependencies in $V(\epsilon)$ account for the fact that a transient state of $A(0)$ may in fact provide a bridge between ergodic classes of $A(0)$ at slower time scales, and thus the "membership" of this transient state must be split in an ϵ-dependent way among the classes it couples.

8

There are several important features of this result. First, the computations at each successive time scale are performed on increasingly aggregated processes as in [Courtois 1977] but unlike [1]. Secondly, the result has a strong graph-theoretic flavor in which one can work solely with the integer orders of the transition rates of $A(\epsilon)$ to determine what the aggregated classes will be and what the structures of $V(\epsilon)$ will be. That is, using only simple integer arithmetic we can determine which elements of U and $V(\epsilon)$ are nonzero and what the orders are of the nonzero elements of $V(\epsilon)$, thereby making the structural computations extremely robust. Finally, a key technical fact used extensively in this development is another "no-cancellation condition", namely the fact that all transition rates between states are nonnegative. The flows of probability mass along two different paths from one state to another therefore add, so that leading-order terms are never canceled.

We feel that the results in [16] represent an important breakthrough, and in fact they have already led to a number of additional results. In particular, we have developed [25, 30] a corresponding aggregation procedure for discrete-time FSMP's. The interesting aspect of this result is that all time scales other than the fastest are described by continuous-time FSMP's. Also, we have developed [25, 31] aggregation results for a large class of continuous-time finite-state semi-Markov processes that go well beyond any other results in the literature. In particular, in our work we have allowed both the transition probabilities and the holding time distributions to be perturbed. By restricting attention to distributions with rational Laplace transforms we are able to use the so-called method of stages to use our FSMP

9

result in order to prove the validity of a hierarchial approximation. Important aspects of this work are (1) the continued use of a no-cancellation condition although the "flow" rates arising from the method of stages are not guaranteed to be positive or even real; (2) the fact that the form of the holding time distribution may lead to a non-transient state at one time scale being split between two aggregates at the next scale -- a form of behavior that cannot occur in an FSMP.

There are three other extensions of this work on which some results have been obtained. In [25, 32, 44] we present some initial results on applying our FSMP results to analyze the reliability of a fault-tolerant system that incorporates an automatic fault detection and reconfiguration system. An important question for such systems is the effect fault detection performance characteristics such as false alarms, missed alarms, and detection delays have on overall reliability. Variations in such parameters can be viewed as changes in the orders of particular transition rates in the FSMP describing the overall system. In [25, 32, 44] we examine a relatively simple problem of this type. Using the fact that our results allow us to identify time scale structure by examination of integer orders of transition rates, we identify particular orders for certain of these rates that lead to overall reliability (as measured by the order of the transition rate from an aggregate state representing "working" to one representing "not working") of maximal order.

The second extension that we have considered is to extend the method in [16] to broader classes of systems of the form of (2.1). In particular, the no-cancellation condition and its flow interpretation suggest possible generalizations. The one we have begun to pursue [25, 36] is to the class of

10

positive systems, i.e., systems for which x(t) is guaranteed to stay in the positive orthant if it begins there. Positive systems can also be represented in a graphical manner, and while the structure of these systems can be far richer than that for FSMP's, we have been able to obtain some results already. In particular, an extension to compartmental models has been obtained. Also, note that not all positive systems will satisfy the MSSNS condition (for example, (2.6) describes a positive system). However, it is possible to determine if a positive system has MSSNS by simple graphical means.

The third extension we have addressed [47] has been motivated by the analysis of flexible manufacturing or inspection and testing systems. In part the work in [47] is a direct application of the methods of [16, 25, 32] to models describing such applications and in particular to the identification of the relationships among certain rates that lead to particular aggregated structures. These applications did, however, lead to one new theoretical result motivated by the fact that in many applications key variables often take the form of <u>counts</u> of particular sets of transitions (such as those modeling completion of a part or of an inspection). An important observation is that at fast enough time scales, these transitions occur as discrete events. However, at slower scales, the states involved in these transitions may be aggregated, and thus the count of "internal transitions" among states contained in an aggregate state must be modeled as a random variable. An asymptotically accurate method for doing this, building in part on results from renewal theory, is developed in [47]. We believe these results will be of significant value in the investigation of control problems for such processes.

11

The final portion of research in this area has dealt with the analysis of control problems associated with the model

$$\dot{x}(t) = A(\epsilon)x(t) + B(\epsilon)u(t) \qquad (2.13)$$

In [9, 20] we present some results on time scale modification, i.e., on modifying the invariant factors of $A(\epsilon)$ by application of feedback of the form

$$u(t) = K(\epsilon)x(t) \qquad (2.14)$$

In our more recent work in this area [34, 35] we have focused on a detailed examination of the controllability structure of (2.13) and its discrete-time counterpart. The key to this analysis is the Smith decomposition of the controllability matrix

$$\mathscr{C}(\epsilon) = [B(\epsilon) \vdots A(\epsilon)B(\epsilon) \vdots \ldots \ldots \vdots A^{n-1}(\epsilon)B(\epsilon)] \qquad (2.15)$$

The invariant factors of this matrix determine the "orders of controllability" of the system, and the Smith decomposition itself allows us to identify a standard form for such systems: order ϵ^0 – controllable states are those that are in the range of $\mathscr{C}(0)$, order ϵ^1 – controllable states are those that are either driven directly by $u(t)$ through an order ϵ gain in $B(\epsilon)$ or have an order ϵ coupling with the order ϵ^0 – controllable states -- i.e.,

$$\dot{x}_1 = A_{11}(\epsilon)x_1(t) + B_1(\epsilon)u(t) + \sum_{i \geq 2} A_{1i}(\epsilon)x_i(t)$$

$$\dot{x}_2 = A_{22}(\epsilon)x_2(t) + \epsilon A_{21}(\epsilon)x_1(\epsilon) + \epsilon B_2(\epsilon)u(t) + \sum_{i \geq 3} A_{2i}(\epsilon)x_i(t)$$

$$\dot{x}_3 = A_{33}(\epsilon)x_3(t) + \epsilon A_{32}x_2(\epsilon) + \epsilon^2 A_{31}x_1(\epsilon) + \epsilon^2 B_3(\epsilon)u(t) + \sum_{i \geq 4} A_{3i}(\epsilon)x_i(t)$$

$$\qquad (2.16)$$

These results allow us to develop asymptotic methods for pole placement via high-gain feedback. In addition, we have some initial results relating

12

the invariant structure of (2.1) with Willems notions of almost-invariance for unperturbed systems [Willems 1981, 1982].

III. SINGULAR SYSTEMS

Our recent research in this area, as documented in [17, 19, 22, 24, 27-29, 40-43], has focused, for the most part, on the class of <u>two-point boundary-value descriptor systems</u> (TPBVDS's):

$$Ex(k+1) = Ax(k) + Bu(k) \qquad (3.1)$$

$$v = V_i x(0) = V_f x(N) \qquad (3.2)$$

Note that E and A may both be singular, so that (3.1) allows one to model a large class of noncausal systems. For this reason, it is natural to analyze this model together with the general boundary condition (3.2). Models of this type and their extension to more than one independent variable frequently arise in the description of spatial or space-time phenomena. Examples range from discretized versions of equations describing electromagnetic fields or gravitational anomalies, to models for distributed systems such as flexible structures, to models that are used as the basis for solving problems in computational vision such as motion estimation and shape from shading (see, in particular, [Rougée 1987] in which the connection between this last class of problems and boundary-value models is made explicit).

Motivated by the wealth of potential signal and image processing applications, we began our investigation in this area with the study of estimation problems for (3.1), (3.2) and also for a particular class of 2-D models (i.e., models with two independent variables) [17, 19, 22]. In particular, in [19] we analyze the problem of estimating $x(k)$ in (3.1), (3.2)

14

given the interior observations

$$y(k) = Cx(k) + r(k) \quad , \quad k\epsilon[1,K-1] \qquad (3.3)$$

and the boundary measurements

$$y_b = W_i x(0) + W_f x(N) + r_b \qquad (3.4)$$

Using the method of complementary models (see [19], [Adams, et al, 1984] and [Weinert and Desai, 1981]) we derived a generalized Hamiltonian form for the optimal estimator:

$$\begin{bmatrix} E & -BQB' \\ 0 & -A' \end{bmatrix} \begin{bmatrix} \hat{x}(k+1) \\ \hat{\lambda}(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C'R^{-1}C & -E' \end{bmatrix} \begin{bmatrix} \hat{x}(k) \\ \hat{\lambda}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ C'R^{-1}y(k) \end{bmatrix}$$

$$(3.5)$$

with appropriate boundary conditions.

Two points of importance in this specification are that (a) the optimal estimator itself is a TPBVDS; and (b) in the standard causal system case $(E = V_i = I, V_f = 0)$ (3.5) reduces to the usual Hamiltonian form for the optimal smoother. The first point raises the question of finding methods for solving these implicit equations, while the latter suggests a possible approach to their solution. In particular, as discussed in [Kailath and Ljung, 1982] and [Adams, et al, 1984], in the causal case it is possible to block-diagonalize or triangularize the Hamiltonian dynamics, yielding a variety of smoothing algorithms including those of Mayne and Fraser and of Rauch, Tung, and Striebel. The specification of the tranformations needed to obtain such algorithms leads directly to Riccati equations, whose properties can in turn be directly related to properties of the original system (e.g. reachability and observability) and of the estimator (e.g. its error covariance and stability).

15

Motivated by this line of research for causal systems, we began an analogous investigation for the estimator (3.5) for a TPBVDS. As described in [19], the possible singularity of E and A makes this a more complex problem. In particular, the approach that exactly parallels one used in the causal case does not work for many TPBVDS's. On the other hand, we have discovered two new generalized Riccati equations

$$\theta = A'(E\theta^{-1}E' + BQB')^{-1}A + C'R^{-1}C \qquad (3.6)$$

$$\Phi = A(E'\Phi^{-1}E + C'R^{-1}C)^{-1}A' + BQB' \qquad (3.7)$$

which, if solutions exist, then provide the basis for block diagonalization of (3.5). In [40, 41] we present some results on the existence and uniqueness of solutions to such equations and their relation to properties of reachability, observability, and stability. To do this, of course, it is necessary to define and study system-theoretic concepts such as these for TDBVDSs, and it was this necessity that led to the extensive set of results described in [24, 27-29, 40, 42, 43] and which we now discuss.

In [24, 27-29, 40, 42, 43] we describe the results of our research to date in developing a system theory for TPBVDS's. O·r line of investigation has been strongly motivated by the work of Krener [1980, 1985a, 1985b] who has investigated the class of standard (i.e. not descriptor) continuous-time boundary-value systems

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (3.8)$$

$$v = V_i x(0) + V_f x(T) \qquad (3.9)$$

Part of our work has paralleled that of Krener, with notable differences because of the potential singularity of both E and A. In addition, our interest in the smoothing problem and in particular in its efficient solution

16

has led us to investigate other topics such as stability and recursive solutions for TPBVDS's.

One of the basic results derived in [27] and used heavily throughout our work is the following. Suppose that (zE-A) is a regular pencil (i.e., its determinant is not identically zero). Then it is possible to transform (3.1), (3.2) so that E and A commute. Well-posedness then is equivalent to the invertibility of $(V_i E^N + V_f A^N)$. In this case, we can always put the system in normlized form, so that

$$V_i E^N + V_f A^N = I \qquad (3.10)$$

$$\alpha E + \beta A = I \qquad (3.11)$$

for some pair of real numbers α and β. As discussed in [27], equation (3.11) greatly simplifies many results connected with TPBVDS's. For example, there is a much simpler statement of a generalized Cayley-Hamilton theorem for E and A in this case, and this in turn leads to simpler reachability and observability results than were available previously.

As in Krener's development, we have explored two notions of recursion for TPBVDS's, namely inward from and outward toward the boundaries, and for each there are associated concepts of reachability and observability. In particular, in [27] we define an inward process $z_i(k,\ell)$, $k < \ell$, which plays a role similar to the state of a causal system in that it represents the boundary condition (rather than initial condition) propogated inward to k and ℓ from 0 and N using the intervening input values (i.e., u(j) for $0 \le j < k$, $\ell \le j < N$). The outward process $z_o(k,\ell)$, on the other hand, summarizes all that one needs to know about the input between k and ℓ in order to determine x outside the interval. While these processes are similar in spirit to those of

17

Krener, the possible singularity of E and A leads to some differences and some additional complexity. For example, in Krener's context z_o represents the difference between the actual value of x at one end of an interval and the value predicted for x at that point given x at the other end of the interval and assuming zero input inside the interval. In our context, we cannot in general predict in either direction, and therefore a modified definition must be developed.

As indicated previously, there are two pairs of notions of reachability and observability. Strong reachability refers to the ability to drive $z_o(k,\ell)$ to any desired value, while weak reachability deals with $z_i(k,\ell)$. Strong observability, on the other hand, refers to the ability to determine $z_i(k,\ell)$ based on observations of u and y between k and ℓ, while weak observability corresponds to our ability to determine $z_o(k,\ell)$ based on knowledge of u and y outside the interval $[k,\ell]$. In [27] we derive conditions for each of these properties and in particular provide justification for the terminology adopted. In addition, we also describe several methods for the efficient solution of TPBVDS's. In particular, one method, which is similar in spirit to two-filter solutions to smoothing problems, involves the simultaneous outward-recursive computation of z_o and inward-recursive computation of z_i. The solution x can then be computed from these quantities. A second solution method, similar in form to the serial structure of the Rauch-Tung-Striebel algorithm, consists first of the outward-recursive computation of z_o, followed by the direct inward-recursive computation of x.

As is the case for causal systems, many of the results for TPBVDS's are simplified and can be carried farther for the class of stationary TPBVDS's,

18

i.e., systems as in (3.1), (3.2) but for which the effect u(k) has on x(ℓ) is a function only of $\ell - k$. In contrast to the causal case, (3.1), (3.2) is not stationary for arbitrary choices of the constant matrices A, B, V_i, and V_f. In [24, 28] we define the class of stationary TPBVDS's as the set of models in (3.1), (3.2) for which V_i and V_f each commute with E and A and for which

$$\ker(E^n) \subset \ker(V_i) \quad , \quad \ker(A^n) \subset \ker(V_f) \tag{3.12}$$

(where n = dim x). As discussed in [24, 28] the technical condition (3.12) which can always be imposed with a modification in system behavior only near the boundaries (and is always true if E and A are invertible) provides some additional regularity and, in particular, allows us to view (3.1), (3.2) as the restriction of a TPBVDS defined on a larger interval. This definition of stationarity also allows us to obtain simplified conditions for weak reachability and observability (the conditions for strong reachability and observability are simple even for nonstationary systems) and to characterize minimal realizations in a fashion that is exactly analogous to Krener's results. In particular, a stationary TPBVDS is a minimal such realization of a given weighting pattern if and only if the system is weakly reachable and observable and the kernel of the strong observability matrix (i.e., the set of "strongly unobservable states") is contained in the range of the strong reachability matrix (i.e., the set of "strongly reachable states"). Also, as in [Krener 1985a], two minimal realizations need not be related only by a similarity transformation, thanks to some flexibility that may exist in the choice of V_i and V_f.

In addition to the result just cited, it is also possible to carry out additional investigations for stationary TPBVDS's. In particular, in [24, 29]

19

we consider two definitions of stability for stationary TPBVDS's. Perhaps the more interesting of the two requires the boundary value v to have an asymptotically vanishing effect on the process x at points _ar from the boundary as the boundaries recede toward $\pm \infty$. (Compare this with the notion of stability for causal systems in which we require the effect of the initial conditions to vanish asymptotically.) The conditions for a stationary TPBVDS to be stable involve both the eigenstructure of $\{E,A\}$ **and** the structure of V_i and V_f. In particular, it is always possible to perform transformations on (3.1) so that

$$E = \begin{bmatrix} I & 0 & 0 \\ 0 & A_b & 0 \\ 0 & 0 & I \end{bmatrix} \quad , \quad A = \begin{bmatrix} A_f & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & U \end{bmatrix} \qquad (3.13)$$

(this is a variation on the Kronecker form for a regular pencil [Van Dooren, 1979]), where A_f and A_b both have all their eigenvalues inside the unit circle and U has all its eigenvalues on the unit circle. In this case the conditions of stationarity require that V_i, V_f also be block diagonal, i.e.,

$$V_i = \text{diag}(V_{i1}, V_{i2}, V_{i3}) \quad , \quad V_f = \text{diag}(V_{f1}, V_{f2}, V_{f3}) \qquad (3.14)$$

Stability, in the sense just defined, then is equivalent to the absence of the third blocks in (3.13), (3.14) (i.e., $|zE-A|$ must be nonzero on the unit circle) together with the invertibility of V_{i1} and V_{f2}. Roughly speaking, the first block of (3.13) corresponds to modes that have stable propogation as k increases and the invertibility of V_{i1} requires that all of these modes be constrained at k = 0. Note that this does **not** mean that these modes are causal since V_{f1} need not be zero. A similar interpretation can be given for the second block.

Another topic investigated in [24, 29] is stochastic stationarity. Specifically, consider a stationary TPBVDS (3.1), (3.2) where v is a zero-mean random vector with covariance Q and u(k) is a zero-mean white noise sequence, independent of v, with variance I. This system is stochastically stationary — i.e., the correlation matrix $\mathcal{E}[x(k)x(\ell)']$ depends only on $k - \ell$ — if and only if Q satisfies the generalized Lyapunov equation

$$EQE' - AQA' = V_i BB'V_i' - V_f BB'V_f' \qquad (3.15)$$

The constant covariance, P, of x(k) in this case satisfies a second generalized Lyapunov equation

$$EPE' - APA' = V_i E^N BB'(V_i E^N)' - V_f A^N BB'(V_f E^N)' \qquad (3.16)$$

Also, in this case it is possible to derive a second-order matrix TPBVDS (analogous to one derived by Krener in his study) whose solution yields the correlation matrix for x.

Finally, as in the causal case, there are results relating Lyapunov equations, stochastic stationarity, and stability, although at present the theory is not complete. In the causal case we know that if a system is reachable from the noise, then stability is equivalent to the existence of a positive definite solution to the system's Lyapunov equation, and this solution represents the initial state covariance that leads to a stationary state process. For TPBVDS, stability is equivalent to the existence of a positive definite solution to (3.16) if the system is strongly reachable. However, even in this case, there may or may not exist a Q satisfying (3.15), so the relationship of stability and stochastic stationarity is not as simple as in the causal case. In fact, if the system is only weakly reachable, it is possible for x to be stochastically stationary even if the system is not stable. A complete clarification of these points remains for the future.

21

IV. SYSTEMS SUBJECT TO DISCRETE EVENTS

The general theme of this portion of our research has been the development of estimation and detection algorithms for several classes of systems subject to discrete events. The area of discrete-event dynamics is one that is presently undergoing a dramatic increase in attention by the research community, as it has been recognized that many estimation and control problems for systems and processes of great complexity have a definite discrete flavor. Major questions that arise in this context include: (1) what kinds of models and theories should be developed? and (2) how do we develop methods capable of dealing with the complexity of many of these problems? This latter question provided the principal motivation for the research described in Section II on multiple time scale/aggregation methods for certain classes of discrete-state processes.

Our work to date in this last portion of our research has been motivated by the first question. Specifically, the concept of a system with discrete events is so broad, it is possible to imagine a large number of alternate mathematical settings that might be candidates for exploration. Consequently, one must look carefully at the potential applications in order to choose meaningful formulations. This has been our approach and in particular we have focused to date on two problem areas and have recently initiated efforts in a third. The first of these, failure detection in dynamic systems, is perhaps the simplest discrete-event problem, as one is interested in detecting individual, isolated events in ordinarily continuously-evolving systems. Our

second research direction, the development of suboptimal distributed estimation algorithms for coupled finite-state processes with applications in electrocardiogram (ECG) analysis, involves considerably more complex event tracking problems and exposes a number of important issues in the monitoring of complex discrete-event processes. Finally, the third area in which we have begun research is the development of system-theoretic concepts for discrete-event systems described by nondeterministic models of the type introduced by Wonham (see, for example, [Vaz and Wonham 1986]).

Our work in failure detection has had three components described in [38], [33], [14], and [23]. One of the major problems in practical failure detection is robustness. Indeed one can argue that this problem is even more challenging in the failure detection context than in the control context since in the failure detection problem one is typically trying to generate signals that are maximally sensitive to some effects (failures) and minimally sensitive to others (model uncertainties). This issue motivated the research described in [38] on the generation of robust redundancy relations for failure detection.

A redundancy relation or parity vector for a perfectly known linear system

$$x(k+1) = Ax(k) + Bu(k) \qquad (4.1)$$

$$y(k) = Cx(k) + Du(k) \qquad (4.2)$$

is a vector v so that

$$v^T \begin{bmatrix} C \\ CA \\ \vdots \\ CA^s \end{bmatrix} = 0 \qquad (4.3)$$

23

for some s $\geq$ 0. As described in [38] one can use v to construct a linear combination of the (s + 1) most recent values of the output and input that will be identically zero if (4.1), (4.2) are precisely correct. Such _parity checks_ can then be used to detect discrepancies between the actual data and that predicted by the model.

Clearly any model uncertainty or noise will contribute to this discrepancy, reducing the value of a parity check for discriminating between normal and failed behavior. In [38] we investigate the problem of maximizing this discrimination capability taking uncertainty and noise into account. For example, suppose that the parameters of the A, B, C, D are uncertain and in particular can take on one of N sets of values indexed by i. Then a criterion capturing the desire to keep the left-hand side of (4.3) small over all possible model parameters is the following:

$$J = \sum_{i=1}^{N} \| v^T Z_i \|^2 \tag{4.4}$$

where

$$Z_i = \begin{bmatrix} C_i \\ C_i A_i \\ \vdots \\ C_i A_i^s \end{bmatrix} \tag{4.5}$$

As discussed in [38], minimizing (4.4), or its generalization to consider a set of several orthogonal parity vectors, can be accomplished through a singular value decomposition of

$$Z = \begin{bmatrix} Z_1 \vdots Z_2 \vdots \ldots \vdots Z_N \end{bmatrix} \tag{4.6}$$

Specifically the singular values of Z indicate the level of robustness of corresponding parity checks. For example, the left singular vector

24

corresponding to the smallest singular value of Z is the most robust parity check. Several important variations on this problem are also considered in [38]. In particular, it is possible to define a statistical version of (4.4) in order to incorporate both the effects of noise and the relative magnitude of the state variables as measured by the state covariance. Also, it is possible to formulate a similar problem in which we want the parity checks to be _large_ when a failure occurs:

$$J = \sum_{i=1}^{N} \|v^T z_i\|^2 - \sum_{i=N+1}^{2N} \|v^T z_i\|^2 \qquad (4.7)$$

where the values of the index i from N + 1 through 2N correspond to the uncertain system dynamics when a particular failure has occured.

While the results in [38] are of significance, they fall short of the complete robust failure detection theory we would like to develop. Specifically, the discrete, parametric specification of model uncertainty (the discrete aspect of which can be relaxed) is restrictive. In particular it would be desirable to have a robustness theory that can handle model uncertainty specified in terms of frequency response error bounds. Also, there is the issue of _designing_ the actual failure detection residual generation system. In particular, the method in [38] determines a set of parity relations, which, as discussed in [Chow and Willsky 1984], can then be used in a number of ways. For example, one method of residual generation consists of simply computing the finite window parity checks determined by these relations. For a variety of reasons, such as noise rejection and enhancement of failure effects for detection and discrimination from other failures, it may be preferable to generate "closed-loop" residuals based on

25

the dynamic models specified by the parity relations -- i.e. to design Kalman filters or observers based on the dynamic relationships specified by the parity relations.

These observations provided the motivation for the investigation described in [33]. While this paper does not deal with the robustness issue, it does establish a linear system-theoretic framework for the design of failure detection systems and in particular makes clear the connections with the geometric and frequency domain theories of linear systems. The specific formulation used in [33] is the following

$$\dot{x}(t) = Ax(t) + Bu(t) + \sum_{i=1}^{k} L_i m_i(t) \qquad (4.8)$$

$$y(t) = Cx(t) \qquad (4.9)$$

where the matrix L_i models the way in which the ith failure mode affects the dynamics and $m_i(t)$ is an arbitrary waveform modeling the actual ith failure time history (see [33] for examples of how various sensor and actuator failures can be modeled in this way). The objective then is to design a residual generation system

$$\dot{w}(t) = Fw(t) - Ey(t) + Gu(t) \qquad (4.10)$$

$$r_i(t) = M_i w(t) - H_i y(t) + K_i u(t) \quad , \ i = 1, \ldots, p \qquad (4.11)$$

so that $r_i(t) \equiv 0 \ \forall i$ if there is no failure and also so that the jth failure mode affects only a subset of the residual vectors, specifically $r_k(t)$, $k \epsilon \Omega_j$. where the coding sets $\Omega_j \subseteq \{1, \ldots, p\}$ are distinct, i.e. $\Omega_i \neq \Omega_j$ for $i \neq j$. If this can be accomplished, then failure detection and identification reduces to a determination of the set of residuals that deviate significantly from zero.

Note that for this approach to be effective, we would also like to make sure that $r_k(t)$, $k\epsilon\Omega_j$ actually do deviate from zero when the jth failure occurs. This is equivalent to the invertibility of the systems defined from $m_j(t)$ as input to the set of signals $r_k(t)$, $k\epsilon\Omega_j$ as output. As discussed in [33], this is too restrictive a condition, and we settle for the less restrictive condition of input observability, i.e. that the columns of the transfer matrix from $m_j(t)$ to the vector $\{r_k(t)$, $k\epsilon\Omega_j\}$ are linearly independent over the field of real numbers (see [33] for a discussion). Also, as developed in the paper, the basic problem on which all of the analysis builds is the fundamental problem in residual generation (FPRG). Specifically, suppose there are only two failure modes in (4.8), i.e. k = 2. The FPRG is to design a residual generator

$$\dot{w}(t) = Fw(t) - Ey(t) + Gu(t) \qquad (4.12)$$

$$r(t) = Mw(t) - Hy(t) + Ku(t) \qquad (4.13)$$

so that the map from (u, m_2) to r is zero and so that the map from m_1 to r is input invertible. There are some strong similarities between this problem and feedback design problems such as decoupling, so it is not suprising that duals of familiar constructs in geometric system theory play an important role. In particular, the concept of an unobservability subspace (see [33]) is crucial. In particular, the FPRG has a solution if and only if the intersection of the range of L_1, and the smallest unobservability subspace containing the range of L_2 is $\{0\}$. Using this basic result, several other problems are solved in [33], including the problem of designing residual generators capable of distinguishing a set of failures under the restrictive assumption that simultaneous failures may occur and the less restrictive situation in which

27

only a single failure can occur. In addition, frequency domain interpretations of these results are given. It is important to note that these results are not simple dualizations of existing results in geometric system theory.

As mentioned previously, the other aspect of our work in discrete-event systems, which is described in [8], [10], [15], and [18], has been on the development of distributed estimation algorithms for a class of coupled finite-state processes. The original motivation for this investigation was to develop a class of event-based models that is appropriate for describing cardiac behavior and for serving as the basis for ECG rhythm analysis. This class of distributed models, however, has much broader potential applicability to other distributed monitoring and situation assessment problems.

As discussed in [15] and [18], each portion of the heart can be viewed as cycling through a set of discrete states corresponding to the electrical events that result in muscle contraction, recovery, and rest. The timing of these events is occasionally and dramatically affected by the occurrence of particular events in other portions of the heart. This structure led us to develop a model structure consisting of a set of N interacting subprocesses each characterized by a state x_i taking values in a finite set. The overall process, with state $\{x_1, \ldots, x_N\}$ is an FSMP possessing, however, a great deal of structure. In particular, conditioned on the present values of all of the subprocesses, the transition behavior of each subprocess is independent of that of the others. Furthermore, while the transition probabilities of subprocess i depend on the values of $\{x_j | j \neq i\}$, there are far fewer values for these probabilities than there are possible sets of states of the other

28

processes. That is, the transition probabilities of subprocess i depend on an interaction variable $h_i(x_j, j \neq i)$ that takes on only a few values. This allows us to model the presence of strong interactions of only a few distinct types among the subsystems, which is a fundamental characteristic of many large-scale systems.

One important aspect of the ECG and many other problems is that the event state is not observed directly. Rather one observes signals that can be viewed as an _encoding_ of particular key transitions in one or more of the subprocesses (in the ECG case these transitions correspond to the initiation of muscular contractions and recoveries resulting in the waveforms seen in the ECG). Consequently, we see that the problem is fundamentally one of finite state estimation or decoding.

A fundamental premise in [18] is that the optimal estimator cannot be implemented (for computational reasons as in ECG analysis or for reasons of geographic separation as in distributed battle management), and that one wishes to design a distributed estimator consisting of interacting processes each responsible for estimating the state of a single subprocess. There are several critical questions that must be dealt with in designing an estimator with this structure. In particular, the processor for a specific subprocess must have some, hopefully reduced, model for the remainder of the system. Also, there typically is a need for processors to exchange information, and the questions that arise are _what_ information should be exchanged and how should the quality of this information be modeled. In [18] we describe one suboptimal but systematic way in which to deal with each of these issues. In particular, since the interaction variable h_i has a set of values of low

29

cardinality, we have developed a method for specifying an FSMP on this set that approximates the behavior of h_i (which is not an FSMP). Also, it is natural for the other processes to provide processor i with an estimate of h_i obtained from the state estimates of the other subprocesses. Since all of the processors are dynamic systems themselves, it is essential that processor i uses a dynamic model for the information it receives from the other processors. Again, one systematic approach to this is described in [18].

Also, as discussed in [18], there is the important issue of performance analysis for such finite state estimators. It is argued that examination of estimation errors at individual points in time is not appropriate in this case, as a small error in the timing of particular events, while yielding large point-to-point errors may actually be of high quality when event sequences are compared. Again one simple approach to capturing such a dynamic error measure is described in [18] (see also [10]), but much more needs to be done in this area.

Furthermore, the distributed nature of our ECG model, with its emphasis on timing and control, has led us to investigate an alternate modeling framework -- stochastic-timed Petri nets -- that offers some advantages in terms of the compactness of the representation and the fact that timing and structural aspects of the model can be described separately. The results of this effort will be described in [40].

Finally there has been a flurry of research on discrete-event dynamic systems modeled as finite state machines or as extended state machines in which time and temporal logic can be incorporated (see, for example, [Vaz and Wonham 1986]). In [45, 46] we present our initial research efforts in this

area.   In particular, we have developed a notion of stability for nondeterministic automata and an associated notion of stabilizability when control is included.   We provide a procedure for determining if such a system is stabilizable and for constructing stabilizing controllers.   A second aspect of our work is motivated by the clear need for aggregate models for such systems if realistic applications are to be considered.   In particular we have developed the notion of a task, consisting of a set of state transitions, described controllers to implement individual tasks, and analyzed the joining of these primitive controllers to implement sets of tasks.   This provides the basis for considering a higher-level description of a discrete-event system in which transitions at the higher level correspond to the completion of tasks at the lower level.

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PUBLICATIONS

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(1)   M. Coderch, A.S. Willsky, S.S. Sastry, and D.A. Castanon, "Hierarchical Aggregation of Linear Systems with Multiple Time Scales," IEEE Trans. on Automatic Control, Vol. AC-28, No. 11, Nov. 1983, pp. 1017-1030.

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34

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